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THE NATURE OF SOUND EMISSION IN THE FORCED VIBRATIONS OF SHELLS IN A COMPRESSIBLE FLUID*

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The nature of sound emission in a compressible fluid outside a sphere completely enclosing a vibrating shell is investigated. The possibility of representing the exact solution of the problem in terms of elementary functions is used. It follows from the formulas obtained that with distance from the sphere the pressure is described by functions decreasing by a power- or polynomial law, with oscillations, where the nature of the oscillations depends on the frequency of the forced vibrations. It is established that the sound intensity is represented in the form of a polynomial with positive coefficients in the square of the reciprocal of the distance from the centre of the sphere. The simple representation for the sound intensity permitted investigation of the nature of the emission from a spherical surface as a function of the frequency and the wave number. The near field is investigated and the influence of the compressible fluid on it is clarified. Simple formulas are obtained for the pressure in the far field.

The problem of forced vibrations of a compressible fluid outside a spherical surface has been examined repeatedly /1-3/. Nevertheless, it is interesting to examine certain questions that are important to the investigation of the nature of sound emission from a surface. In particular, it is clarified that the widespread assumption about the exponential damping of pressure in a fluid during vibrations of a shell therein /4-7/* (*See also Vasil'ev D.G. and Simonov I.V. Asymptotic estimates of the complex frequencies of vibrations of a shell in a fluid. Preprint No.186, Inst. Probl. Mekhan., Akad. Nauk, SSSR, Moscow, 1981.) is not correct.

We investigate the forced vibrations of a compressible fluid outside a sphere S of radius R in which a vibrating shell subjected to a surface load that varies with time according to the harmonic law $e^{-i\omega t}$ is completely enclosed. In the case of a spherical shell the sphere S coincides with the shell. In the case of an arbitrary shell, this sphere is in the closest proximity to the shell (and can coincide with it at individual points).

The fluid vibrations outside the shell are governed completely by its normal displacements which can be found by solving the problem of the combined vibrations of the shell and the fluid. For a shell of complex shape this is by no means a simple problem. In the case of a spherical shell the method of separation of variables enables a solution to be obtained in the form of an expansion in spherical functions /8/. However, there is no need here to examine the complete solution of the problem of shell vibrations in a fluid.

In a spherical coordinate system $\bar{r}, \theta, \varphi$, the fluid vibrations outside a sphere S are determined by the displacement potential $\Phi(\bar{r}, \theta, \varphi)$ (the time coordinate is omitted throughout), which satisfies the Helmholtz equation

$$\Delta\Phi(\bar{r}, \theta, \varphi) + \bar{k}^2\Phi(\bar{r}, \theta, \varphi) = 0, \quad \bar{k}^2 = \omega^2/a^2$$

(ω is the frequency of vibration, and a is the speed of sound in the fluid) under the condition that for $\bar{r} = R$ (R is the radius of the sphere), the derivative of $\Phi(\bar{r}, \theta, \varphi)$ with respect to $\bar{r}$ equals the normal displacements on the sphere S while the Sommerfeld radiation condition is satisfied at infinity.

Separation of variables enables the solution of this problem to be represented as an expansion in spherical functions. We investigate the pressure field in the fluid that corresponds to one term in the expansion. Such an investigation is important in problems of sound emission in connection with resonance phenomena that manifest themselves as a sudden growth of the displacements and pressures at the so-called resonance frequencies. The resonating harmonic yields the fundamental contribution to the magnitudes of the displacement and pressure at such frequencies.

The pressure in a fluid, that corresponds to a specific term in the expansion in spherical harmonics, is represented in the form

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$$p_{nm}(r, \theta, \varphi) = p_n(1)\Psi_n(k, r) P_{nm}(\cos\theta)\cos m\varphi \tag{1}$$
$$\Psi_n(k, r) = r^{-1/2} H_{n+0,5}(kr)/H_{n+0,5}(k), \quad k = \bar{k}R, \quad r = \bar{r}/R$$

The quantity $p_n(1)$, equal to the maximum value of the pressure on the sphere S, depends on parameters characterizing the shell and the fluid. The resonance properties of the shell vibrations in the fluid are reflected by its values. The quantity $P_{nm}(\cos\theta)\cos m\varphi$ yields the angular pressure distribution. The dependence of $p_{nm}(r, \theta, \varphi)$ on the dimensionless distance r is defined by the functions $\Psi_n(k, n)$.

The magnitude of the sound intensity

$$I = |p|^2/(2\rho a)$$

(ρ is the fluid density, and p is the pressure) is of great value for the sound pressure field characteristics.

The sound intensity equals the energy flux transferred by a wave through unit area perpendicular to the wave propagation direction. We obtain the following expression for the sound intensity corresponding tothe pressure field (1):

$$I^{(n, m)} = \frac{1}{2\rho a} |p_n(1)|^2 |\Psi_n(k, r)|^2 |P_{nm}(\cos\theta)\cos m\varphi|^2 \tag{2}$$

The quantity $|p_n(1)|^2/(2\rho a)$ determines the maximum value of the sound intensity on the sphere S while the quantity $|P_{nm}(\cos\theta)\cos m\varphi|^2$ is its anagular distribution.

The functions $\Psi_n(k, r)$, $\Phi_n(k, r) = |\Psi_n(k, r)|^2$ characterizing the dependences of the pressure and sound intensity in the fluid on r, are defined completely just by the values of the parameters k and n. This permits investigation of the nature of sound pressure emission from the sphere S without determining the value of $p_n(1)$, whose value could be found only by the complete solution of the problem of the combined vibrations of the shell and fluid.

Let us transform $\Psi_n(k, r)$, $\Phi_n(k, r)$ by using expressions for the Hankel function of half-integer order in terms of elementary functions.

It should be noted that there is a representation of the solution of the problem under consideration in terms of elementary functions /1, 2/. The results of this paper could also be obtained from relationships presented in /1, 2/ without using Hankel functions.

We have

$$\operatorname{Re}\Psi_n(k, r) = \frac{1}{F_n(k)}\frac{1}{r}[E_n(k, r)\cos k(r-1) + G_n(k, r)\sin k(r-1)] \tag{3}$$
$$\operatorname{Im}\Psi_n(k, r) = \frac{1}{F_n(k)}\frac{1}{r}[-G_n(k, r)\cos k(r-1) + E_n(k, r)\sin k(r-1)]$$
$$\Phi_n(k, r) = \frac{1}{F_n(k)}\frac{1}{r^2}F_n(kr)$$
$$E_n(k, r) = Q_n^{(1)}(kr)Q_n^{(1)}(k) + Q_n^{(2)}(kr)Q_n^{(2)}(k)$$
$$G_n(k, r) = Q_n^{(1)}(kr)Q_n^{(2)}(k) - Q_n^{(2)}(kr)Q_n^{(1)}(k)$$
$$F_n(kr) = [Q_n^{(1)}(kr)]^2 + [Q_n^{(2)}(kr)]^2$$
$$Q_n^{(1)}(x) = \sum_{\nu=0}^{[n/2]} \frac{(-1)^\nu (n+2\nu)!}{(2\nu)!(n-2\nu)!(2x)^{2\nu}}$$
$$Q_n^{(2)}(x) = \sum_{\nu=0}^{[(n-1)/2]} \frac{(-1)^\nu (n+2\nu+1)!}{(2\nu+1)!(n-2\nu-1)!(2x)^{2\nu+1}}$$

It is seen that $\operatorname{Re}\Psi_n(k, r)$, $\operatorname{Im}\Psi_n(k, r)$ are the sum of products of polynomials of degree $n+1$ in $1/r$ and $\cos k(r-1)$ and $\sin k(r-1)$. Therefore, the functions $\operatorname{Re}\Psi_n(k, r)$, $\operatorname{Im}\Psi_n(k, r)$ that characterize the pressure change in r, decreases with r with the oscillations. The nature of the oscillations depends on the dimensionless frequency k.

It is simplest to establish the nature of the sound emission near the sphere on the basis of a study of the dependence of the sound intensity $I^{(n, m)}$ on r, k, n, which is defined by the function $\Phi_n(k, r)$.

Let us clarify the properties of the polynomial $F_n(kr)$ in terms of which the function $\Phi_n(k, r)$ is expressed. We set

$$F_n(x) = 1 + \sum_{j=1}^{n} a_j^{(n)} x^{-2j}$$

Exact formulas are obtained for the coefficients $a_j^{(n)}$. We note that in the numerical determination of these coefficients by squaring and adding the polynomials $Q_n^{(i)}(x)$ $(i = 1, 2)$ in terms of which $F_n(x)$ are expressed, a loss in accuracy occurs for compartively moderate n.

It is found that

$$a_j^{(n)} = A_j B_j^{(n)}, \quad A_1 = 1/2, \quad B_1^{(n)} = n(n+1)$$
$$A_j = \frac{1}{2^{2j}} \frac{2}{(j-1)!} (j+1)(j+2)\ldots(j+(j-1))$$
$$B_j^{(n)} = n(n^2-1^2)(n^2-2^2)\ldots(n^2-(j-1)^2)(n+j) \; (j \geqslant 2)$$

It is hence seen that all $a_j^{(n)} > 0$. It can be seen that their magnitude increases abruptly as the power $1/x^2$ increases.

We note that the function $\Phi_n(k,r)$ and the sound intensity $I^{(n,m)}$ decrease monotonically as r increases. We investigate the behaviour of $\Phi_n(k,r)$ near the sphere S. From a comparison of the coefficients in $F_n(kr)$ for different powers of $1/r^2$ with the free term one, we obtain $a_j^{(n)} k^{2j} = 1$ or $k = \sqrt[2j]{a_j^{(n)}} \; (1 \leqslant j \leqslant n)$. These equations define a series of curves Γ_j in the n, k plane to whose points the values of n, k correspond for which the coefficient of $1/r^{2j}$ is commensurate with unity. It is seen from Fig.1, where the curves Γ_j $(j = 5; 10; 15; 20)$ and Γ_n $(j = n)$ are represented, that these curves fill a certain domain G of the n, k plane fairly compactly, where the curves Γ_j depart fan-like from the curve Γ_n. It is characteristic for the curves Γ_j that as n increases they all approximate to straight lines quite rapidly. This follows from the face that for large n the equations of Γ_j can be represented with $O(n^{-2})$ accuracy in the form

$$j = 1, \quad k = \left(\frac{1}{2}\right)^{1/2} \left(n + \frac{1}{2} - \frac{1}{8n}\right); \quad j = 2, \quad k = \left(\frac{3}{8}\right)^{1/4} \left(n + \frac{1}{2} - \frac{5}{8n}\right)$$
$$j = 3, \quad k = \left(\frac{5}{16}\right)^{1/6} \left(n + \frac{1}{2} - \frac{35}{24n}\right); \ldots$$
$$j = n, \quad k = \frac{2}{e}\left[n + \frac{\ln 2}{2} + \left(\frac{1}{4}\ln^2 2 - \frac{1}{24}\right)\frac{1}{n}\right]$$

For values of n, k from the domain G, the contributions of the separate components to the quantity $F_n(kr)$ are of the same order for r slightly different from unity.

For the domain G_1 of values of n, k located below the domain G, the main contribution to the quantity $F_n(kr)$ for r close to one, is made by the term with the highest power of $1/r^2$, i.e., the term $a_n^{(n)}/(k^{2n} r^{2n})$.

For the domain G_2 of values of n, k located above the domain G, the quantity $F_n(kr)$ is determined by the free term one for r close to unity.

Therefore, in the n, k plane there are three domains with a different nature in the change in sound intensity near the sphere. In the domain G_1 the dependence of the sound intensity around the sphere on r is close to $1/r^{2n+2}$, i.e., is approximately the same as in the case of an incompressible fluid. For these values of n, k the influence of the fluid compressibility near the sphere turns out to be unimportant.

In the domain G_2 the dependence of the sound intensity $I^{(n,m)}$ around the sphere on r is close to $1/r^2$. For these values of n, k, the influence of compressibility is felt in immediate proximity to the sphere S and emission from the sphere penetrates most remotely into the depths of the fluid.

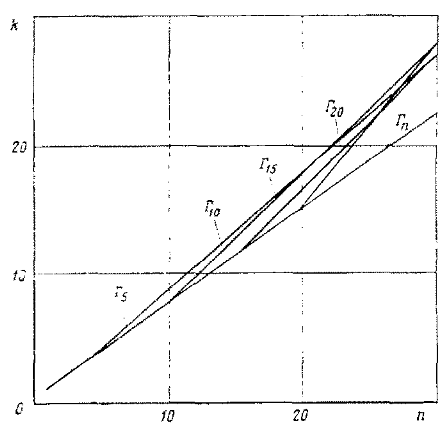

Fig.1

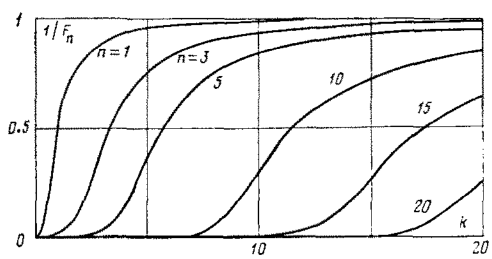

Fig.2

For values of n, k from the domain G the dependence on r near the sphere S is determined by a polynomial of degree $n+1$ in $1/r^2$, all of whose terms are of approximately the same order.

As the distance from the sphere S increases for low and medium frequencies (the domains G and G_1) a gradual increase occurs in the role of the terms with lower powers of $1/r^2$ in the expression for the sound intensity $I^{(n,m)}$. The dependence of the sound intensity on r becomes close to $1/r^2$ at a significant distance from the sphere S.

We obtain simplified formulas for the pressure and sound intensity in the far field for large values of r in the case of low and medium frequencies (the domains G_1 and G) and for any r for high frequencies (the domain G_2). We have

$$p_{nm}(r, \theta, \varphi) = p_n(1) \frac{1}{\sqrt{F_n(k)}} \frac{1}{r} e^{i(k(r-1)-\chi)} P_{nm}(\cos\theta)\cos m\varphi \tag{4}$$

$$I^{(n,m)}(r, \theta, \varphi) = \frac{1}{2\rho a} |p_n(1)|^2 \frac{1}{F_n(k)} \frac{1}{r^2} |P_{nm}(\cos\theta)\cos m\varphi|^2 \tag{5}$$

$$\operatorname{tg}\chi = Q_n^{(2)}(k)/Q_n^{(1)}(k)$$

We note that $p_n(1)$ in (4) and (5) is complex in value. Moreover, the factors $1/\sqrt{F_n(k)}$ and $1/F_n(k)$, whose values impinge substantially on the magnitudes of the sound pressure and intensity determined by the simplified formulas, enter into (4) and (5).

For each n the values of $1/F_n(k)$ increase from zero to one as k increases (Fig.2). As n increases, the frequency range increases for which $1/F_n(k)$ takes quite small values. The sound pressure and intensity in the far field are insignificant at these frequencies. Assuming the dependences of $1/F_n(k)$ on k, the frequencies can be determined for each n for which the emission is penetrating in nature. For these frequencies the values of $1/F_n(k)$ do not differ radically from unity.

In conclusion, we note that representations in elementary functions are obtained for the sound pressure and intensity which enable them to be determined in the whole mass of fluid outside the spherical surface depending on their amplitude values on the sphere. Three domains of values are established for the parameters n, k with a different near-field nature. It is found that the sound intensity near the sphere varies as $1/r^{2n+2}$ for low frequencies, as $1/r^2$ for high frequencies, and according to a polynomial law of degree $n+1$ in $1/r^2$ for medium frequencies. Note that the frequency ranges depend on the value of the wave number.

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